

On the cost-complexity of multi-context systems

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Abstract

Multi-context systems provide a powerful framework for modelling information-aggregation systems featuring heterogeneous reasoning components. Their execution can, however, incur non-negligible cost. Here, we focus on *cost-complexity* of such systems. To that end, we introduce *cost-aware multi-context systems*, an extension of non-monotonic multi-context systems framework taking into account costs incurred by execution of semantic operators of the individual contexts. We formulate the notion of cost-complexity for consistency and reasoning problems in MCSs. Subsequently, we provide a series of results related to gradually more and more constrained classes of MCSs and finally introduce an incremental cost-reducing algorithm solving the reasoning problem for definite MCSs.

1 Introduction

Deployment of large-scale sensor networks and exploitation of heterogeneous databases concentrating various kinds of information about the real world opens new horizons in real-time information aggregation and processing systems. Sensed information can be instantly cross-validated, fused, reasoned about and further processed in real-time, so as to provide constant and up-to-date situational awareness for people, systems, or organisations. Such knowledge-intensive information-aggregation systems find applications in a range of industrial domains, from marine traffic monitoring to applications in supporting assisted living environments. With the grow of computing power, however, it's rather the resource costs incurred by running such systems, which pose an obstacle to their deployment, rather than the time-complexity of their execution. Such costs can include network bandwidth, electricity, battery life, but also direct financial costs of accessing 3rd party databases, or utilisation of costly communication channels, such as satellite data-links.

Non-monotonic multi-context systems (MCS) introduced by Brewka & Eiter (2007) are a powerful framework for interlinking heterogeneous knowledge sources. The framework traces its origins back to the seminal work by Giunchiglia & Serafini (1994) on multi-language hierarchical logics. A multi-context system comprises a number of

knowledge bases, contexts, each encapsulating a body of information, together with a corresponding mechanism for its semantic interpretation and reasoning with it. The flow of information among the knowledge bases is regulated by a set of bridge rules of the form “if L is true according to the semantics of the context i and ..., then L' needs to be taken into account by the context j .” Due to the abstraction from the particularities of the internal semantics of the individual contexts and the focus on analysis of the information flow between them in a rigorous manner, multi-context systems provide a suitable abstraction for modelling a wide range of information-aggregation systems, such as those mentioned above.

Consider an information-aggregation system aiming at surveillance and anomaly detection in maritime traffic. Such a system would source a range of data elements from a deployed large-scale sensor network including radars or antennae and would cross-validate the information with that stored in local or remote databases providing data about vessel types, owners, etc. Similarly, an information-aggregation system supporting an assisted-living environment would continuously sense data about well-being of patients from a range of sensors and fuse it with relevant health records, etc. A typical query to such systems could aim at detection whether a vessel, or a patient might need operator’s attention, such as whether a ship might be involved in suspicious activities, or whether a patient is possibly in a life threatening condition. Querying such physical information sources can, however, be relatively costly, while the time-complexity of reasoning with such components plays a lesser role.

We model such information-aggregation systems as multi-context systems as follows. The contexts correspond to information-processing agents and information-source adapters, each encapsulating a fragment of the information-fusion functionality of the system according to some internal semantics with an attached cost. The contexts are linked to each other by bridge rules prescribing the information flow within the aggregation process, typically from low-level sensory evidence and raw information retrieved from various databases to higher-level hypotheses a user might be interested in.

To facilitate such multi-context systems, here we propose the framework of *cost-aware multi-context systems*, an extension of the generic framework of non-monotonic multi-context systems (Brewka & Eiter, 2007). Subsequently, after introducing the notion of cost-complexity of algorithms over MCSs, in a series of analyses we provide worst-case cost-complexity upper bounds for problems of consistency and reasoning with general, definite and acyclic MCS. We conclude our discourse with an algorithm for incremental reasoning in definite MCSs, version of which we also implemented and deployed in METIS, a prototype system for maritime traffic surveillance (Hendriks & van de Laar, 2013; Velikova *et al.*, 2014).

2 Cost-aware multi-context systems

We build the framework of cost-aware multi-context systems as an extension of the original non-monotonic multi-context systems by Brewka & Eiter (2007).

Definition 1 (logic suite). A *logic suite* $L = (\text{KB}, \text{BS}, \text{ACC}, \text{cost})$ is composed of the following components:

KB is the set of well-formed finite knowledge bases of L . We assume that each element of KB is a finite set and that $\emptyset \in \text{KB}$;

BS is the set of possible finite belief sets;

$\text{ACC} : \text{KB} \rightarrow 2^{\text{BS}}$ is a semantic operator which, given a knowledge base kb , returns a set of sets of acceptable beliefs, each with cardinality polynomial in the size of kb ; and finally

$\text{cost} : \text{KB} \rightarrow \mathbb{R}^+$ is a cost function assigning to each knowledge base $kb \in \text{KB}$, the cost associated with a single execution of the semantic operator ACC over kb . Consequently, $\text{cost}(\text{ACC}) = \max_{kb \in \text{KB}} \text{cost}(kb)$ denotes the maximal cost which can be incurred by invocation of ACC over the knowledge bases of KB .

Relative to the original formulation, the definition above introduces several simplifications. We focus on the subclass of finite non-monotonic multi-context systems, those with finite knowledge bases and bridge rule sets. We also identify the acceptable belief sets returned by the semantic operator ACC with their poly-size kernels (c.f. Brewka & Eiter 2007).

Definition 2 (bridge rule). Let $L = \{L_1, \dots, L_n\}$ be a set of logic suites. An L_i -bridge rule over L with $1 \leq i \leq n$, is of the form

$$s \leftarrow (c_1 : p_1), \dots, (c_j : p_j), \text{ not } (c_{j+1} : p_{j+1}), \dots, \text{ not } (c_m : p_m)$$

where $c_k = 1..n$, $p_k \in S_{c_k}$ is an element of some belief set $S_{c_k} \in \text{BS}_{c_k}$ of L_{c_k} , and for each $kb \in \text{KB}_i$, we have that $kb \cup \{s\} \in \text{KB}_i$.

For a bridge rule r of the above form, $\text{head}(r) = s$ and $\text{body}(r) = \{p_1, \dots, p_m\}$ denote the head and the body of r . We say that literals s and p_1, \dots, p_n occur in the head and the body of r respectively.

Definition 3 (multi-context system). A cost-aware multi-context system (MCS) $M = (C_1, \dots, C_n)$ consists of a collection of contexts $C_i = (L_i, br_i)$, where $L_i = (\text{KB}_i, \text{BS}_i, \text{ACC}_i, \text{cost}_i)$ is a logic suite and br_i is a set of L_i -bridge rules over $\{L_1, \dots, L_n\}$.

The sets of knowledge bases and belief sets effectively determine the input/output interface languages for a context C_i . To let a context process a new information, a new element needs to be added to its knowledge base. Conversely, retrieving information from a context corresponds to inspecting its belief set.

In contrast to the original definition, we do not require a context to have an initial knowledge base, as such “default” input to the semantic operator can be contained directly in its semantics, i.e., not necessarily $\text{ACC}(\emptyset) \neq \emptyset$.

Definition 4 (notation). We say that r is a bridge rule of a MCS $M = (C_1, \dots, C_n)$ iff there exists $i = 1..n$, such that the set of bridge rules br_i of the context C_i contains r , i.e., $r \in br_i$. We also say that M contains r . Similarly, M contains a set of bridge rules R if it contains every rule $r \in R$. Finally, for convenience, let $\mathcal{R}(M) = \bigcup_{i=1}^n br_i$ denote the set of bridge rules of M .

Definition 5 (belief state and satisfied rules). Let $M = (C_1, \dots, C_n)$ be a MCS. A *belief state* is a tuple $S = (S_1, \dots, S_n)$, such that each S_i is an element of BS_i . We define set operations on belief states as the corresponding set operations on their respective belief set projections.

A bridge rule r of the form introduced in Definition 2 is said to be *satisfied* in a belief state S iff for all $i = 1..j$ we have $p_i \in S_{c_i}$ and for all $k = j + 1..m$ we have $p_k \notin S_{c_k}$.

Definition 6 (equilibrium). A belief state $S = (S_1, \dots, S_n)$ of a MCS $M = (C_1, \dots, C_n)$ with $C_i = (L_i, br_i)$ is an *equilibrium* of M iff for all $i = 1..n$ we have that $kb_i = \{head(r) \mid r \in br_i \text{ is a rule satisfied in } S\}$ and for each i , we have $S_i \in ACC_i(kb_i)$.

3 Reasoning with cost-aware MCSs

The following definition of consistency and reasoning problems reiterates the original one by Brewka & Eiter.

Definition 7 (consistency and reasoning problems). Given a MCS $M = (C_1, \dots, C_n)$, the problem of M 's *consistency* equals to deciding whether there exists an equilibrium $S = (S_1, \dots, S_n)$ of M .

Given an element p , a *query*, the problem of *brave reasoning* is to decide whether there is an equilibrium $S = (S_1, \dots, S_n)$ of M , such that $p \in S_i$ for some $i = 1..n$. We also say that S entails p . Finally, the problem of *cautious reasoning* is to decide whether all equilibria of M entail p .

Due to the opaqueness of the individual contexts in a MCS, an algorithm for deciding a problems of consistency, brave, or cautious reasoning, would in general need to search for a solution by testing various knowledge bases as inputs to contexts, executing their internal semantic operators, and finally check whether the outputs are coherent with the knowledge bases. Informally, the cost incurred by a run of such a computation over the input MCS corresponds to the sum of the costs associated with the series of invocations of the semantic operators of the individual contexts.

Definition 8 (cost-complexity). Let \mathcal{A} be a *deterministic* algorithm taking as an input a MCS M and computing a particular belief state state S of M as its output, along the way employing the semantic operators of the individual contexts. Given a series of semantic operator invocations $ACC_{c_1}, \dots, ACC_{c_m}$ performed during \mathcal{A} 's execution, $Cost_{\mathcal{A}}(M) = \sum_{i=1}^m cost(ACC_{c_i})$ denotes the sum of the costs of their corresponding invocations. We also say that $Cost_{\mathcal{A}}(M)$ is a cost-complexity of \mathcal{A} 's computation on M .

The *worst-case cost-complexity* of \mathcal{A} is a function $Cost_{\mathcal{A}} : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ defined by

$$Cost_{\mathcal{A}}(n, m) = \max\{Cost_{\mathcal{A}}(M) \mid M \in \overline{M}_{n,m}\}$$

where $\overline{M}_{n,m}$ is a set of all MCSs composed of precisely n contexts and m bridge rules. That is, for each $M \in \overline{M}_{n,m}$, we have $M = (C_1, \dots, C_n)$, with each C_i comprising bridge rules br_i and $m = |\mathcal{R}(M)| = \sum_{i=1}^n |br_i|$.

In the restricted case when the number of bridge rules m in an MCS is bounded with respect to the number of its contexts n by some finite factor $k \in \mathbb{N}$, we define $Cost_{\mathcal{A}}(n) = Cost_{\mathcal{A}}(n, k \cdot n)$.

Consider a special class of MCSs with uniform unit cost of execution of all semantic operators of their corresponding contexts. For such MCSs, the notion of cost-complexity of algorithms reduces to the notion of time-complexity in terms of the number of invocations of the context semantic operators.

Definition 9 (context-independent time complexity). A MCS $M = (C_1, \dots, C_n)$ is said to be *uniform-cost* iff for all $i = 1..n$, we have $cost(ACC_i) = 1$, with ACC_i corresponding to C_i .

The context-independent time complexity is defined as $CTime_{\mathcal{A}}(M) = Cost_{\mathcal{A}}(M) = m$. Consequently, the context-independent worst-case complexity of \mathcal{A} is defined as $CTime_{\mathcal{A}}(n, m) = Cost_{\mathcal{A}}(n, m)$ over uniform-cost MCSs with n contexts and m bridge rules. $CTime_{\mathcal{A}}(n)$ and $CTime(n)$ are defined accordingly in relation to $Cost_{\mathcal{A}}(n)$ and $Cost(n)$.

Finally, we analyse the context-independent time complexity and the cost-complexity of the class of general non-monotonic cost-aware multi-context systems.

Proposition 10 (consistency). *Given a uniform-cost MCS $M = (C_1, \dots, C_n)$, an upper bound on the worst-case context-independent time complexity of deciding the consistency problem for M , as well as problems of cautious and brave reasoning w.r.t. M for some query p , we have*

$$CTime(n, m) \leq n \cdot 2^m$$

where m is the number of bridge rules in M .

In the case M is not a uniform-cost MCS, an upper bound on the worst-case cost-complexity of deciding the consistency problem for M , as well as problems of cautious and brave reasoning w.r.t. M for some query p , we have

$$Cost(n, m) \leq c \cdot CTime(n, m)$$

where $c = \max_{i=1..n} cost(ACC_i)$.

Proof. Consider the following algorithmic schema:

1. guess the set of bridge rules R to be satisfied in an equilibrium;
2. construct the knowledge bases kb_1, \dots, kb_n , so that $kb_i = \{head(r) \mid r \in R\}$;
3. execute the individual contexts' semantic operators on the knowledge bases and thus obtain a belief state S ; and finally
4. check whether S is an equilibrium. That is, exactly the rules from R are those satisfied in S .

In general, there are at most 2^m candidate sets of rules to guess in the step 1 of the non-deterministic schema above. For each of them, we need to invoke at most n semantic operators in the step 3, what in turn incurs a cost of at most c per invocation.

For brave and cautious reasoning problems, in the worst case, we need to enumerate all the possible belief states to check whether they are equilibria and additionally whether they entail the query p . Hence, the worst-case cost complexity of reasoning problem equals the one of the consistency problem. \square

The above result is consistent with the complexity analysis of Brewka & Eiter. They show that the time complexity of the decision problems as a step-up over the time complexity of ACC operator: if the context time-complexity is in Δ_k^P , then the time-complexity of deciding the consistency problem lies in Σ_{k+1}^P . We abstract away from the context time-complexity and consider it constant (P), hence the expected step-up corresponds to NP .

4 Definite cost-aware MCS

The cost-complexity characteristics of reasoning with general cost-aware multi-context systems as introduced in the previous section is rather pessimistic. Even brave reasoning incurs in general cost complexity exponential in the size of the information-flow structure of the system. For practical purposes, that can become prohibitive as the size of the system scales. Often, however, information flows of implemented systems feature simpler structure both in terms of the individual contexts, as well as in terms of the underlying flow of information. *Definite cost-aware multi-context systems*, an adaptation of the notion reducible MCSs (Brewka & Eiter, 2007), provide a suitable model for such systems.

Definition 11 (monotonic logic suite). Let $L = (\text{KB}, \text{BS}, \text{ACC}, \text{cost})$ be a logic suite. L is *monotonic* iff

1. $\text{ACC}(kb)$ is a singleton set for every $kb \in \text{KB}$; and
2. $kb \subseteq kb', \text{ACC}(kb) = \{S\}$ and $\text{ACC}(kb') = \{S'\}$ implies $S \subseteq S'$.

Definition 12 (definite MCS). Let $M = (C_1, \dots, C_n)$ be a MCS. We say that M is *definite* iff

1. the logic suites L_1, \dots, L_n corresponding to the contexts C_1, \dots, C_n are monotonic; and
2. none of the bridge rules in any context contains *not*.

Definition 13 (grounded equilibrium). Let M be a definite MCS. $S = (S_1, \dots, S_n)$ is the *grounded equilibrium* of M iff S is the unique set-inclusion minimal equilibrium of M .

Remark 14. Every definite MCS has exactly one unique equilibrium, which is grounded (Brewka & Eiter, 2007).

Algorithm 1 Algorithm for computing the grounded equilibrium of a definite MCS.

Input: a definite MCS $M = (C_1, \dots, C_n)$

Output: returns the grounded equilibrium of M

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1: let  $kb_i^0 \leftarrow \emptyset$  for all  $i = 1..n$  and  $S^0 \leftarrow (\emptyset, \dots, \emptyset)$ 
2:  $k \leftarrow 0$ 
3: repeat
4:    $S^k \leftarrow (S_1^k, \dots, S_n^k)$  with  $S_i^k = \text{ACC}_i(kb_i^k)$ 
5:    $kb^{k+1} \leftarrow (kb_1^{k+1}, \dots, kb_n^{k+1})$  with
      $kb_i^{k+1} = kb_i^k \cup \{head(r) \mid r \text{ is satisfied in } S^k\}$ 
6:    $k \leftarrow k + 1$ 
7: until  $S^k \neq S^{k-1}$ 
8: return  $S^k$ 

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Unsurprisingly, the cost-complexity of reasoning for definite MCS is significantly lower than for general MCS. The following proposition provides the first upper estimate on the worst-case cost-complexity of the consistency problem in definite MCS.

Proposition 15 (consistency). *Given a uniform-cost definite MCS $M = (C_1, \dots, C_n)$, an upper bound on the worst-case context-independent time complexity of deciding the consistency problem for M , and thus also the problems of cautious and brave reasoning w.r.t. M for some query p is*

$$C\text{Time}(n, m) \leq n \cdot m$$

In the case M is not a uniform-cost MCS, an upper bound on the worst-case cost-complexity of deciding the consistency problem for M , we have

$$\text{Cost}(n, m) \leq c \cdot C\text{Time}(n, m)$$

where $c = \max_{i=1..n} \text{cost}(\text{ACC}_i)$.

Proof sketch. Consider Algorithm 1. To compute an equilibrium (and thus decide the consistency problem), all contexts have to be invoked at least once. After each iteration either at least one head literal which was not true in the previous iterations becomes true and does not cease afterwards, or no new head literal is inferred. If the latter is the case, the process can stop. Thus, there are at most m rule heads to become true over at most m steps. In every step, there are at most n context being executed. Hence the upper bound. \square

Corollary 16. *In most instances of implemented systems the number of rules m in a multi-context system will dominate the number of contexts n . Hence the worst-case time-independent complexity would typically be at most quadratic in the number of bridge rules, i.e., $C\text{Time}(n, m) \leq m^2$ and consequently also $\text{Cost}(n, m) \leq c \cdot m^2$.*

Now we turn our attention to the cost-complexity of reasoning in definite MCSs. Since such MCSs have only a single unique equilibrium, the problems of brave and cautious reasoning collapse and in turn we speak only about a reasoning problem. It

turns out, that the cost-complexity is, similarly to the general MCS case, bound by the cost-complexity of deciding the consistency problem, but in many practical cases can be pushed lower. Before introducing the main result in Proposition 21, we first analyse the structure of information flow leading to supporting the individual belief sets in an equilibrium.

Definition 17 (fragmentary MCS). We say that a MCS $M' = (C'_1, \dots, C'_n)$ is a *MCS fragment* of another MCS $M = (C_1, \dots, C_n)$ iff for all context $C'_i = (L'_i, br'_i)$ and $C_i = (L_i, br_i)$ with $i = 1..n$, we have that $L'_i = L_i$ and $br'_i \subseteq br_i$. We also denote $M' \subseteq M$ and say that M *contains* M' . Set operations on MCS fragments are defined as the corresponding set operations on their respective bridge rule sets.

Definition 18 (justification). Let $M = (C_1, \dots, C_n)$ be a definite MCS with a grounded equilibrium $S = (S_1, \dots, S_n)$. A *justification* for a belief set S_i in M is a MCS fragment $M' \subseteq M$, such that

1. M' has a grounded equilibrium $S' = (S'_1, \dots, S'_n)$, with $S'_i = S_i$ and $S'_j \subseteq S_j$ for every $j \neq i$; and
2. there is no other fragment $M'' \subseteq M'$ satisfying the condition 1.

A justification of a belief set S_i in a MCS M corresponds to the minimal set of bridge rules of M which still enable derivation of S_i . Justifications are defined w.r.t. a given equilibrium. Support is a complementary syntactic counterpart to the notion of justification.

Definition 19 (support). Let $M = (C_1, \dots, C_n)$ be a definite MCS. The *input signature* of a context C_i is a set of literals $sig(C_i) = \{p \mid \exists r \in br_i : head(r) = p\}$. An *immediate rule support* of a context $C_i = (L_i, br_i)$ is a set of rules $In \subseteq br_i$, such that for every $p \in sig(C_i)$, there exists a rule $r \in In$ with $head(r) = p$. Finally, a *support* for a context $C_i = (L_i, br_i)$ is a fragment $M' = (C'_1, \dots, C'_n)$ of M with $C'_i = (L'_i, br'_i)$, such that

1. br'_i contains some rule support In of C_i ;
2. for every context C'_j literals of which occur in a body of some bridge rule of M' , M' contains also a support for C'_j . That is, for every rule $r \in br'_j$, if $(c_k : p_k) \in body(r)$, then M' contains a support of C'_j . Finally,
3. for every context C'_j , we have that br'_j is minimal w.r.t. set inclusion, such that the conditions 1 and 2 are satisfied.

$\mathcal{M}(C_i)$ denotes the set of all supports of C_i . Furthermore, $\overline{\mathcal{C}}(C_i) = \{C'_l \mid \exists M' = (C'_1, \dots, C'_n) \in \mathcal{M}(C_i) \text{ and } br'_l \neq \emptyset\}$ and $\overline{\mathcal{R}}(C_i) = \{r \mid \exists M' \in \mathcal{M}(C_i) \text{ and } r \in \mathcal{R}(M')\}$ denote the sets of contexts and rules respectively *supporting* C_i in various supports in M .

Note, there might be several immediate rule supports for a context C_i due to possibly multiple bridge rules with the same head literal. Also, minimality of bridge rule

sets ensures that for each literal p , there is only a single rule r in a support M' with $p = \text{head}(r)$. In turn, there might be multiple supports for a given context in M .

The following proposition relates the syntactic characterisation of sets of rules potentially justifying a given belief set, the support, and the sets of rules serving as an actual justification of the belief set in an already computed belief set.

Proposition 20. *Let $M = (C_1, \dots, C_n)$ be a definite MCS with a grounded equilibrium $S = (S_1, \dots, S_n)$. For every belief set S_i of S and each of its justifications M_{just} , there exists a support $M_{supp} \subseteq M$ of C_i , such that $M_{just} \subseteq M_{supp}$ and all the bridge rules of M_{just} are satisfied in S .*

Proof sketch. For a justification $M_{just} = (C_1^j, \dots, C_n^j)$ of a belief set S_i with contexts $C_i^j = (L_i^j, br_i^j)$, we construct a fragment M_{supp} of M which will also be a support of C_i in M .

Firstly, for every context C_j of M , either there exists an immediate rule support In of C_j , such that $In = br_j^{just}$, or we find an immediate rule support In of C_j , such that $br_j^{just} \subseteq In$. Existence of such a suitable immediate rule support In of C_j is ensured by the minimality of br_j^{just} (c.f. Condition 2 of Definition 18), which ensures that for each $p \in \text{sig}(C_j)$, there's at most one rule r satisfied in M_{just} with $p = \text{head}(r)$, and the construction of immediate supports of a context, which require not only minimality of In (c.f. Condition 3 of Definition 19), but also a full coverage of $\text{sig}(C_j)$. We construct a fragment M_{supp} by simply extending each br_j^{just} with one of such suitable immediate rule supports. In a consequence, we have that $M_{just} \subseteq M_{supp}$ and M_{supp} automatically satisfies the conditions on being a support of C_i stipulated in Definition 19. \square

A corollary of the above proposition is that for deciding a reasoning problem over an MCS M and a query p , we only need to consider the contexts and rules relevant to p . That is, those which support the context C_i to which p belongs, because every possible justification of a p -entailing belief set of C_i must be a subset of some support of C_i . Hence, due to monotonicity of contexts of M , we can simply compute the equilibrium of a fragment related to a union of all the possible supports of C_i and check whether it entails p .

Proposition 21. *Let $M = (C_1, \dots, C_n)$ be a uniform-cost definite MCS and let p be a query, an element of some belief set of C_i . An upper bound on the worst-case context-independent time complexity of brave reasoning w.r.t. M for some query p is*

$$CTime(n, m) \leq |\overline{\mathcal{C}}(C_i)| \cdot |\overline{\mathcal{R}}(C_i)| \leq n \cdot m$$

Consequently for the case where M is not a uniform-cost MCS, an upper bound on the worst-case context-independent time complexity of brave reasoning w.r.t. M for some query p is

$$Cost(n, m) \leq c \cdot CTime(n, m) \leq c \cdot |\overline{\mathcal{C}}(C_i)| \cdot |\overline{\mathcal{R}}(C_i)| \leq c \cdot n \cdot m$$

where $c = \max_{C_j \in \overline{\mathcal{C}}(C_i)} \text{cost}(\text{ACC}_j)$.

Corollary 22. *Similarly to the observation in Corollary 16, in most instances of implemented systems the number of rules m in supports of a given context C_i will dominate the number of contexts n , the worst-case time-independent complexity of reasoning problem would typically be at most quadratic in terms of the number of rules of the relevant support. In the worst-case, though, a support can include all the bridge rules of the original MCS.*

5 Acyclic definite MCS

Now we turn our attention to multi-context systems which do not contain cycles in the information flow structure induced by their bridge rules. While relatively simplistic, the class of acyclic MCSs is practically important. Many knowledge-intensive systems-of-systems and information-aggregation applications feature a hierarchical structure with raw information sources at the bottom and gradually more and more abstract and higher-level information-processing components towards the top. The hierarchical structure of such systems is dictated by the fact, that such systems capture the knowledge of human experts in a given domain. Typically, the structure of domain knowledge articulated by such experts tends to be relatively simple and hierarchical too, as such structures are easier to understand and manipulate for humans.

Definition 23 (stratified definite MCS). Let $M = (C_1, \dots, C_n)$ be a definite MCS with contexts $\mathcal{C}(M) = \{C_1, \dots, C_n\}$. A decomposition $\mathfrak{S} = \mathfrak{S}_0, \dots, \mathfrak{S}_m$ of M with $\mathfrak{S}_i \subseteq \mathcal{C}(M)$ and $\mathfrak{S}_k \cap \mathfrak{S}_l = \emptyset$ for every $k, l = 1..n$ is a *stratification* of M iff for each bridge rule $r \in br_i$ of a context $C_i \in \mathfrak{S}_k$ each of its body literals $(j : p) \in body(r)$ corresponds to a context $C_j \in \mathfrak{S}_l$ with $l < k$.

We say that a stratification $\mathfrak{S} = \mathfrak{S}_0, \dots, \mathfrak{S}_m$ of a MCS M is *compact* iff there is no other stratification $\mathfrak{S}' = \mathfrak{S}'_0, \dots, \mathfrak{S}'_{m'}$ of M , such that there is a context C of M with $C \in \mathfrak{S}_i$ and $C \in \mathfrak{S}'_j$, while at the same time $j < i$.

A definite MCS M is said to be *stratified*, or *acyclic*, iff there exists a stratification of M .

In stratified definite MCSs the information flows unidirectionally from contexts without any bridge rules in the stratum \mathfrak{S}_0 (information sources), upwards to higher-level contexts up to those in the top-most stratum for which there are no bridge rules in the MCS containing elements of their belief states in their respective bodies. Thanks to stratification, we also straightforwardly have that all the contexts involved in supports of a given context C belong to lower strata than C does and at the same time there are no bridge-rule dependencies among the context of a single stratum. This insight leads to the following cost complexity result.

Proposition 24. *Given a uniform-cost stratified definite MCS $M = (C_1, \dots, C_n)$, an upper bound on the worst-case context-independent time complexity of deciding the consistency problem for M , as well as the reasoning problem w.r.t. M for some query p is*

$$CTime(n, m) \leq n$$

Algorithm 2 Algorithm computing the grounded equilibrium in a definite stratified MCS.

Input: a stratified definite MCS $M = (C_1, \dots, C_n)$

Output: returns S , the grounded equilibrium of M

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1: compute  $\mathfrak{S} = \mathfrak{S}_0, \dots, \mathfrak{S}_l$ , the compact stratification of  $M$ 
2:  $S \leftarrow (S_1, \dots, S_n)$  with  $S_i \leftarrow \emptyset$ 
3: for  $k = 1..l$  do
4:   for each  $C_i \in \mathfrak{S}_k$  do
5:      $kb_i \leftarrow \{head(r) \mid r \in br_i \text{ is applicable in } S\}$ 
6:     update  $S$  with  $S_i = ACC_i(kb_i)$ 
7:   end for
8: end for
9: return  $S$ 

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In the case M is not a uniform-cost MCS, an upper bound on the worst-case cost-complexity of deciding the consistency problem for M , as well as the reasoning problem w.r.t. M for some query p , we have

$$Cost(n, m) \leq c \cdot CTime(n, m) = c \cdot n$$

where $c = \max_{i=1..n} cost(ACC_i)$.

Proof. Algorithm 2 computes the grounded equilibrium of a MCS M , hence it decides the consistency problem of M . Its soundness and completeness follows inductively from the following induction hypothesis: Let S_{c_1}, \dots, S_{c_k} be belief sets of contexts C_{c_1}, \dots, C_{c_k} involved in body literals of a union of all immediate rule supports of a context C_i in a stratum \mathfrak{S}_k . Given a knowledge base $kb_i = \{head(r) \mid r \in br_i \text{ and } r \text{ is satisfied w.r.t. } S_{c_1}, \dots, S_{c_k}\}$, $ACC_i(kb_i) = \{S_i\}$ is a singleton belief set C_i corresponding to the grounded equilibrium of M . The algorithm evaluates every context of M exactly once, hence its worst-case cost-complexity equals the number of contexts of M . \square

6 Incremental reasoning in definite MCS

The discourse in previous sections followed the structure of standard results from logic programming reflected in the framework of multi-context systems. Generally, the cost-complexity results followed the known results on computational time complexity of computation of models of logic programs. In this section, we build upon the concepts introduced above and introduce an incremental approach for computing equilibria of cost-aware multi-context systems, which specifically focuses on reduction of cost-complexity of such computation.

By exploiting the structure of a given MCS with respect to a given query, the actual cost-complexity of solving the reasoning problem can be often improved upon. Before introducing an actual algorithm reducing the number of context evaluations while computing a solution to a reasoning problem, let us first introduce a notion of a fragment

depending on a context. The concept is complementary to the context support (Definition 19), but instead of considering contexts and rules necessary for deriving a belief set of a context C_i , a *fragment depending on a context* is a fragmentary MCS in which computation of belief sets of all other contexts is influenced, *depends* on the context C_i . In other words, change in a belief set of C_i can potentially enforce a change in the input knowledge base and thus also a change in the output belief set of the depending contexts.

Definition 25 (fragment depending on a context). Let $M = (C_1, \dots, C_n)$ be a definite MCS. We say that a fragment $M' = (C'_1, \dots, C'_n)$ of M *depends* on a context C_i of M iff for all contexts C'_j we have that $(i : p) \in \text{body}(r)$ with $r \in \text{br}_j$ implies that $r \in \text{br}'_j$ and at the same time M' contains a fragment M'' *depending* on C'_j .

Given a set of contexts $\mathcal{C} = C_{i_1}, \dots, C_{i_k}$, we say that M' *depends precisely* on \mathcal{C} iff for each context $C_{i_j} \in \mathcal{C}$ it contains the fragment of M depending on C_{i_j} , while at the same time M' is minimal such w.r.t. set inclusion on the sets of bridge rules.

For convenience, we define an *empty fragment* as a fragment with all bridge rule sets empty. We also say, that a context $C = (L, \text{br})$ is *valid* iff $\text{br} \neq \emptyset$.

Proposition below provides an insight into computation required for “extending” a grounded equilibrium of a MCS fragment to a grounded equilibrium of its extension by another fragment.

Proposition 26. Let $M = (C_1, \dots, C_n)$ be a definite MCS and M' be its fragment depending on a context C_i with a grounded equilibrium $S' = (S'_1, \dots, S'_n)$. Let also M_{supp} be a fragment of M depending precisely on the set of contexts valid in $M_{\text{diff}} = M \setminus M'$. Recall, the set of bridge rules corresponding to the i -th context of M_{diff} is defined as $\text{br}_i^{\text{diff}} = \text{br}'_i \setminus \text{br}_i$ and M_{supp} depends precisely on the contexts C_{i_1}, \dots, C_{i_k} , such that $\text{br}_{i_k}^{\text{diff}} \neq \emptyset$.

The grounded equilibrium $S = (S_1, \dots, S_n)$ of M can be computed as a union $S = S' \cup S_{\text{supp}}$, where S_{supp} is a grounded equilibrium of M_{supp} .

Proof sketch. M can be decomposed into two fragments disjoint w.r.t. their bridge rules: M' and $M \setminus M'$. In turn, for every context C_i of M , we have that either

1. its projection in $M \setminus M'$ is a valid context, i.e., there exists at least one bridge rule in its projection in $M \setminus M'$; or
2. its projection in $M \setminus M'$ features an empty set of bridge rules, but the context depends on another context for which 1 is the case, or
3. all its bridge rules were already contained in M' and the same holds for all the contexts it is supported by, depends on.

In the first case, the belief set of the context C_i needs to be recomputed, since a new literal needs to be possibly added to the context’s knowledge base. Due to monotonicity of logic suites in a definite MCS, it is ensured that the resulting belief set will be a (non-strict) superset of the original one. Should that be the case, also all the contexts depending on it need to be recomputed as well, regardless whether their bridge rules

completely belonged to M' , or not. In the second case, by the same argument the context's belief set needs to be recomputed as well. Finally, when the context's bridge rules completely belonged to M' and it does not depend on any context which needs to be recomputed, its belief set equals the corresponding belief set in the grounded equilibrium of M . \square

Corollary 27. *Let $M_1, M_2 \subseteq M$ be fragments of a MCS M . A grounded equilibrium S_1 of M_1 can be extended to a grounded equilibrium $S_{1,2}$ of a union $M_{1,2} = M_1 \cup M_2$ as $S_{1,2} = S_1 \cup S_{add}$, where S_{add} is a grounded equilibrium of a fragment of $M_{1,2}$ depending precisely on the contexts valid in $M_1 \setminus M_2$.*

Corollary 27 is a straightforward reformulation of Proposition 26. We conclude the discourse by exposing Algorithm 3 for incremental reasoning in definite MCSs and exploiting the corollary. It first computes all the supports of the context the query corresponds to and then iteratively selects them one by one and incrementally constructs the fragmentary grounded equilibrium of the input MCS. If during the computation p is derived as an equilibrium of some of the fragments, due to monotonicity of logic suites in a definite MCS, by necessity, the equilibrium of M must also entail p .

Applied to stratified definite MCS, the algorithm could be further improved by selecting the cheapest fragment of \mathcal{M} . That is, one with the lowest equilibrium computation estimate for its corresponding M_{supp} fragment. In stratified definite MCSs that cost corresponds to the sum of costs of the contexts which need to be recomputed.

Note, the applicability of the algorithm is constrained to MCS information-flow structures with relatively small overlaps between different supports of the context deriving the query answer. In presence of high redundancy among contexts, that is almost all contexts depend on almost all the other, the algorithm will recompute the whole MCS too often, though. This is, however, seldom the case in implemented systems, such as the maritime traffic surveillance system METIS (Hendriks & van de Laar, 2013).

7 Discussion and final remarks

The motivating premise underlying the presented work is that MCSs are a suitable model for design, implementation and analysis of deployed knowledge-intensive systems-of-systems featuring heterogeneous components. Here, we extend the generic model of multi-context systems with the notion of a cost of executing semantic operators of the individual contexts. The idea is to facilitate system scalability in terms of the incurred costs, be it computational costs, bandwidth, or even financial expenses. Our focus on cost-complexity, rather than computational-complexity of algorithms also presents a novel view on design and deployment of information-aggregation and reasoning systems.

We introduced a series of worst-case complexity results for general, definite and stratified MCSs. Some of these results could be still improved upon by taking inspiration from e.g., (Bairakdar *et al.*, 2010), where the authors perform an ear-decomposition of possibly cyclic MCSs, so as to streamline distributed computation of their equilibria. Another inspiration could be to exploit the results stemming from the results

Algorithm 3 Incremental reasoning for definite MCSs.

Input: a definite MCS $M = (C_1, \dots, C_n)$ and a query p corresponding to a context C_i

Output: returns true iff the grounded equilibrium of M entails p

```
1: compute the set  $\mathcal{M}$  of fragments of  $M$  supporting  $C_i$ 
2:  $M_{done} \leftarrow$  an empty fragment of  $M$ 
3:  $S \leftarrow (S_1, \dots, S_n)$  with  $S_i = \emptyset$ 
4: repeat
5:   select  $M' \in \mathcal{M}$ 
6:    $\mathcal{M} \leftarrow \mathcal{M} \setminus \{M'\}$ 
7:   construct  $M_{supp}$ , a fragment of  $M_{done} \cup M'$  precisely
     depending on the valid contexts of  $M' \setminus M_{done}$ 
8:    $S_{supp} \leftarrow$  compute the equilibrium of  $M_{supp}$ 
9:    $S \leftarrow S \cup S_{supp}$ 
10:   $M_{done} \leftarrow M_{done} \cup M'$ 
11:  if  $S$  entails  $p$  then return true
12: until  $\mathcal{M} = \emptyset$ 
13: return false
```

by Gottlob, Pichler, & Wei (2006) and analyse the information-flow graph induced by bridge rules of an MCS, in order to exploit the results relating the time-complexity of computation with such systems to the tree-width of the information-flow graph.

We already implemented the incremental algorithm for deciding reasoning problems in the context of our work on METIS, a system-of-systems aiming at maritime traffic surveillance and risk assessment of ships sailing in busy coastal waters, such as in the Dutch Exclusive Economic Zone. We describe the system and its reconfiguration component based on ideas described above in a separate submission (Velikova *et al.*, 2014).

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